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The topological AC effect on non-commutative phase space

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Abstract. The Aharonov–Casher (AC) effect in non-commutative (NC) quantum mechanics is studied. Instead of using the star product method, we use a generalization of Bopp's shift method. After solving the Dirac equations both on non-commutative space and non-commutative phase space by the new method, we obtain corrections to the AC phase on NC space and NC phase space, respectively.

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1 Introduction

In recent years, there has been increasing interest in the study of physical effects on non-commutative space; apart from field theory, non-commutative quantum mechanics has recently attracted much attention, because the effects of the non-commutativity of the space may become significant in extreme situations such as at the string scale or at the TeV level and higher. Many papers have been devoted to the study of various aspects of quantum mechanics on NC space with the usual (commutative) time coordinate [1–9]. For example, the topological Aharonov–Bohm (AB) phase on NC space and even on NC phase space have been studied in [2–7]. In this work, different from the method employed in [8], we develop a new method to obtain corrections to the topological phase of the AC effect both on NC space and NC phase space, where we know that on a commutative space the line spectrum does not depend on the relativistic nature of the dipoles. The article is organized as follows: in Sect. 2, by using the Lagrangian formulation, we discuss the AC effect on commutative space in $2+1$ dimensions. In Sect. 3, the AC effect on a non-commutative space is studied, and the correction to the AC phase on NC space is obtained. The AC effect on non-commutative phase space will be discussed in Sect. 4, and a most generalized formula for the holonomy on NC phase space is given in this section; finally, some remarks are made in the last section.

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2 Description of the AC effect on $2+1$ commutative space-time

To begin with, let us give a brief review of the AC effect on $2+1$ commutative space-time. The Lagrangian for a neutral particle of spin half with an anomalous magnetic dipole moment μ_{m}, interacting with an electromagnetic field, has the form

$$L=\bar{\psi}\mathrm{i}\gamma^{\mu}\partial_{\mu}\psi-m\bar{\psi}\psi-\frac{1}{2}\mu_{\mathrm{m}}\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}\,. \tag{1}$$

The last term in the Lagrangian is responsible for the AC effect.

We restrict ourselves to the case that the particle moves on a plane (say the x–y plane); then the problem can be treated on $2+1$ space-time. We use the following conventions for the $2+1$ dimensional metric $g_{\mu\nu}$ and the antisymmetric tensor $\epsilon_{\mu\nu\alpha}$:

$$g_{\mu\nu}=\mathrm{diag}(1,-1,-1)\quad\text{and}\quad\epsilon_{012}=+1\,. \tag{2}$$

Different from the use of 2×2 matrices satisfying the $2+1$ dimensional Dirac algebra, we will use three four-component Dirac matrices that can describe spin up and down in the z direction for a particle and for its antiparticle. In $2+1$ dimensions these Dirac matrices satisfy the following relation [10]:

$$\gamma^{\mu}\gamma^{\nu}=g^{\mu\nu}-\mathrm{i}\gamma^{0}\sigma^{12}\epsilon^{\mu\nu\lambda}\gamma_{\lambda}\,. \tag{3}$$

A particular representation is

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} \mathrm{i}\sigma_2 & 0 \\ 0 & -\mathrm{i}\sigma_2 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} \mathrm{i}\sigma_1 & 0 \\ 0 & \mathrm{i}\sigma_1 \end{pmatrix}. \tag{4}$$

Now the interaction term in the Lagrangian can be written

$$\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} = F^{\mu\nu}\gamma^0\sigma^{12}\epsilon_{\mu\nu\lambda}\bar{\psi}\gamma^{\lambda}\psi\,, \tag{5}$$

with

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 \\ E^1 & 0 & -B^3 \\ E^2 & B^3 & 0 \end{pmatrix}, \tag{6}$$

where E^i and B^i are the electric and magnetic fields, respectively. The indices “1” and “2” indicate the coordinates on the x–y plane along the x and y direction. The index “3” indicates the z direction. The Lagrangian now can be written

$$L = \bar{\psi}\mathrm{i}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi - (1/2)\gamma^0\sigma^{12}\mu_{\mathrm{m}}\epsilon_{\mu\alpha\beta}F^{\alpha\beta}\bar{\psi}\gamma^{\mu}\psi\,. \tag{7}$$

By using the Euler–Lagrange equation, the Dirac equation of motion for a neutral spin-half particle with a magnetic dipole moment μ_{m} is

$$\left(\mathrm{i}\gamma_{\mu}\partial^{\mu} - (1/2)\gamma^0\sigma^{12}\mu_{\mathrm{m}}\epsilon_{\mu\alpha\beta}F^{\alpha\beta}\gamma^{\mu} - m\right)\psi = 0\,, \tag{8}$$

and the solution will have the form

$$\psi = \mathrm{e}^{-\frac{\mathrm{i}}{2}\gamma^0\sigma^{12}\mu_{\mathrm{m}}\int^x \epsilon_{\mu\alpha\beta}F^{\alpha\beta}\mathrm{d}x^{\mu}}\psi_0\,, \tag{9}$$

where ψ_0 is the solution for the electromagnetic field free case. The phase in (9) is called the AC phase; we write it as follows:

$$\phi_{\mathrm{AC}} = -\frac{1}{2}\gamma^0\sigma^{12}\mu_{\mathrm{m}}\int^x \varepsilon_{\mu\alpha\beta}F^{\alpha\beta}\mathrm{d}x^{\mu}\,. \tag{10}$$

The above AC phase is the general AB phase for a neutral spin-half particle passing through an electromagnetic field. If we consider the situation of the standard AC configuration [11, 12], i.e. a particle moving in a plane under the influence of a pure electric field produced by a uniformly charged infinitely long filament perpendicular to the plane, then we have

$$\begin{aligned}\phi_{\mathrm{AC}} &= -\gamma^0\sigma^{12}\mu_{\mathrm{m}}\int^x \varepsilon_{0ij}F^{0i}\mathrm{d}x^j \\ &= \gamma^0\sigma^{12}\mu_{\mathrm{m}}\int^x (\hat{\mathbf{k}}\times\mathbf{E})\cdot\mathrm{d}\mathbf{x} \\ &= \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}\int^x (\boldsymbol{\mu}_{\mathrm{m}}\times\mathbf{E})\cdot\mathrm{d}\mathbf{x}\,,\end{aligned} \tag{11}$$

where $\hat{\mathbf{k}}$ is the unit vector in the z direction, and we assume that the magnetic dipole moment is always along this direction, i.e. $\boldsymbol{\mu}_{\mathrm{m}} = \mu_{\mathrm{m}}\hat{\mathbf{k}}$.

3 The AC effect on non-commutative space

On a non-commutative space the coordinate and momentum operators satisfy the following commutation relations (we take $\hbar = c = 1$ unit):

$$[\hat{x}_i, \hat{x}_j] = \mathrm{i}\Theta_{ij}\,, \quad [\hat{p}_i, \hat{p}_j] = 0\,, \quad [\hat{x}_i, \hat{p}_j] = \mathrm{i}\delta_{ij}\,, \tag{12}$$

where Θ_{ij} is an element of an antisymmetric matrix, which is very small compared to the energy scale. It represents the non-commutativity of the NC space; $\hat{x}_i$ and $\hat{p}_i$ are the coordinate and momentum operators on the NC space.

Just like the static Schrödinger equation on NC space [7], the Dirac equation (8) for a neutral spin-half particle with a magnetic dipole moment μ_{m} on NC space can be written

$$\left(\mathrm{i}\gamma_{\mu}\partial^{\mu} - (1/2)\gamma^0\sigma^{12}\mu_{\mathrm{m}}\epsilon_{\mu\alpha\beta}F^{\alpha\beta}\gamma^{\mu} - m\right) * \psi = 0\,, \tag{13}$$

i.e., simply replace the usual product with a star product (Moyal–Weyl product), and the Dirac equation on the usual commuting space will change into the Dirac equation on NC space. The star product of two functions is defined by

$$\begin{aligned}(f*g)(x) &= \mathrm{e}^{\frac{\mathrm{i}}{2}\Theta_{ij}\partial_{x_i}\partial_{x_j}}f(x_i)g(x_j) \\ &= f(x)g(x) + \frac{\mathrm{i}}{2}\Theta_{ij}\partial_i f\partial_j g\Big|_{x_i=x_j} + \mathcal{O}\left(\Theta^2\right)\,;\end{aligned} \tag{14}$$

here $f(x)$ and $g(x)$ are two arbitrary functions.

Some features of the AC effect on non-commutative space have been studied in [8] by using the star calculation, but it is yet useful to study it again by using the method given in [7], i.e. through a generalized Bopp shift, and this method can easily be generalized to NC phase space, as will be discussed in the next section.

On a NC space the star product can be replaced by a Bopp shift, i.e. the star product can be changed into an ordinary product by shifting the coordinates x_{μ}:

$$\hat{x}_{\mu} = x_{\mu} - \frac{1}{2}\Theta_{\mu\nu}p^{\nu}\,. \tag{15}$$

Now, let us consider the non-commutative Dirac equation (13). To replace the star product with an ordinary product, equivalent to the Bopp shift, the $F_{\mu\nu}$ must, up to first order of the NC parameter Θ, be shifted by

$$F_{\mu\nu} \to \hat{F}_{\mu\nu} = F_{\mu\nu} + \frac{1}{2}\Theta^{\alpha\beta}p_{\alpha}\partial_{\beta}F_{\mu\nu}\,. \tag{16}$$

Then the Dirac equation for the AC problem on NC space has the form

$$\left(\mathrm{i}\gamma_{\mu}\partial^{\mu} - (1/2)\gamma^0\sigma^{12}\mu_{\mathrm{m}}\epsilon_{\mu\alpha\beta}\hat{F}^{\alpha\beta}\gamma^{\mu} - m\right)\psi = 0\,. \tag{17}$$

Therefore, on NC space, the AC phase has the form

$$\begin{aligned}\hat{\phi}_{\mathrm{AC}} &= -\frac{1}{2}\gamma^0\sigma^{12}\mu_{\mathrm{m}}\int^x \varepsilon_{\mu\alpha\beta}\hat{F}^{\alpha\beta}\mathrm{d}x^\mu \\ &= -\frac{1}{2}\gamma^0\sigma^{12}\mu_{\mathrm{m}}\int^x \varepsilon_{\mu\alpha\beta}F^{\alpha\beta}\mathrm{d}x^\mu \\ &\quad -\frac{1}{4}\gamma^0\sigma^{12}\mu_{\mathrm{m}}\int^x \epsilon_{\mu\alpha\beta}\Theta^{\sigma\tau}p_\sigma\partial_\tau F^{\alpha\beta}\mathrm{d}x^\mu\,. \end{aligned} \tag{18}$$

This is the general AC phase for a neutral spin-half particle moving in a general electromagnetic field.

In the standard AC configuration, the momentum on NC space can be written[1]

$$p_l = mv_l + (\mathbf{E}\times\boldsymbol{\mu})_l + \mathcal{O}(\theta)\,, \tag{19}$$

where $\boldsymbol{\mu}=\mu_{\mathrm{m}}\boldsymbol{\sigma}$; next we insert (19) into (18) and notice that

$$F^{\alpha\beta}\longrightarrow F^{0i} \quad \text{and} \quad \Theta^{ij}=\theta\epsilon^{ij},\ \Theta^{0\mu}=\Theta^{\mu 0}=0\,. \tag{20}$$

Then we have

$$\hat{\phi}_{\mathrm{AC}} = \phi_{\mathrm{AC}} + \delta\phi_{\mathrm{NCS}}\,, \tag{21}$$

where ϕ_{AC} is the AC phase on commuting space given by (11), and the additional phase $\delta\phi_{\mathrm{NCS}}$, related to the non-commutativity of the space, is given by

$$\begin{aligned}\delta\phi_{\mathrm{NCS}} &= -\frac{1}{2}\gamma^0\sigma^{12}\mu_{\mathrm{m}} \\ &\quad\times\int^x \epsilon_{\mu 0 i}\theta\epsilon^{\alpha\beta}[mv_\alpha + (\mathbf{E}\times\boldsymbol{\mu})_\alpha]\partial_\beta F^{0i}\mathrm{d}x^\mu \\ &= \frac{1}{2}\gamma^0\sigma^{12}\mu_{\mathrm{m}}\theta\epsilon^{ij} \\ &\quad\times\int^x [k_j + (\mathbf{E}\times\boldsymbol{\mu})_j]\left(\partial_i E^2\mathrm{d}x^1 - \partial_i E^1\mathrm{d}x^2\right)\,, \end{aligned} \tag{22}$$

where $k_j = mv_j$.[2] If we have the spin of the neutral particle along the z direction (namely, we can choose $\boldsymbol{\mu}=\sigma_3\mu_{\mathrm{m}}\hat{\mathbf{k}}$), then our result here will be exactly the same as the result given in [8], where the tedious star product calculation has been used.

4 The AC effect on non-commutative phase space

We have discussed the AC effect on NC space, where space–space do not commute but with momentum–momentum commuting. Bose–Einstein statistics in non-commutative quantum mechanics requires both space–space and momentum–momentum non-commutativity. We call the NC space with non-commuting momenta: NC phase space. To study the physics on NC phase space is very important. On NC phase space, the commutation relation in (12) for the momentum $\hat{p}_i$ should be replaced with

$$[\hat{p}_i,\hat{p}_j] = \mathrm{i}\bar{\Theta}_{ij}\,, \tag{23}$$

where $\bar{\Theta}$ is the antisymmetric matrix, and its elements represent the non-commutativity of the momenta. The Dirac equation (8) on NC phase space can also be written

$$\left(-\gamma_\mu p^\mu - (1/2)\gamma^0\sigma^{12}\mu_{\mathrm{m}}\epsilon_{\mu\alpha\beta}F^{\alpha\beta}\gamma^\mu - m\right)*\psi = 0\,, \tag{24}$$

but now the star product in (24) defines

$$\begin{aligned}(f*g)(x,p) &= \mathrm{e}^{\frac{\mathrm{i}}{2\alpha^2}\Theta_{ij}\partial_i^x\partial_j^x + \frac{\mathrm{i}}{2\alpha^2}\bar{\Theta}_{ij}\partial_i^p\partial_j^p} f(x,p)g(x,p) \\ &= f(x,p)g(x,p) + \frac{\mathrm{i}}{2\alpha^2}\Theta_{ij}\partial_i^x f\partial_j^x g\Big|_{x_i=x_j} \\ &\quad + \frac{\mathrm{i}}{2\alpha^2}\bar{\Theta}_{ij}\partial_i^p f\partial_j^p g\Big|_{p_i=p_j} + \mathcal{O}\left(\Theta^2\right)\,, \end{aligned} \tag{25}$$

where $\mathcal{O}(\Theta^2)$ stands for second and higher order terms in Θ and $\bar{\Theta}$. The star product in the Dirac equation on NC phase space can be replaced by the usual product from these two steps: first we need to replace x_i and p_i with a generalized Bopp shift as follows:

$$\begin{aligned} x_\mu &\to \alpha x_i - \frac{1}{2\alpha}\Theta_{\mu\nu}p_\nu\,, \\ p_\mu &\to \alpha p_\mu + \frac{1}{2\alpha}\bar{\Theta}_{\mu\nu}x_\nu\,, \end{aligned} \tag{26}$$

and also we need the partner of the shift in (16) on NC phase space, which is as follows:

$$F_{\mu\nu}\to\hat{F}_{\mu\nu} = \alpha F_{\mu\nu} + \frac{1}{2\alpha}\Theta^{\alpha\beta}p_\alpha\partial_\beta F_{\mu\nu}\,. \tag{27}$$

The Dirac equation (24) then reads

$$\begin{aligned}\Big\{ &-\alpha\gamma^\mu p_\mu - \frac{1}{2\alpha}\gamma^\mu\bar{\Theta}_{\mu\nu}x_\nu - (1/2)\gamma^0\sigma^{12}\mu_{\mathrm{m}}\epsilon_{\mu\alpha\beta} \\ &\times\left[\alpha F^{\alpha\beta} + \frac{1}{2\alpha}\Theta^{\tau\sigma}p_\tau\partial_\sigma F^{\alpha\beta}\right]\gamma^\mu - m\Big\}\psi = 0\,. \end{aligned} \tag{28}$$

[1] In the standard AC configuration the Hamiltonian on commuting space has the form [11, 12]

$$H = \frac{1}{2m}\boldsymbol{\sigma}\cdot(\mathbf{p}-\mathrm{i}\mu_{\mathrm{m}}\mathbf{E})\boldsymbol{\sigma}\cdot(\mathbf{p}+\mathrm{i}\mu_{\mathrm{m}}\mathbf{E})\,.$$

In the region $\nabla\cdot\mathbf{E}=0$ (if the particle does not reach the charged filament, this is always true), the above equation can be recast in the form

$$H = \frac{1}{2m}(\mathbf{p}-\mathbf{E}\times\boldsymbol{\mu})^2 - \frac{\mu^2E^2}{2m}\,;$$

then we have for the velocity operator

$$v_l = \frac{\partial H}{\partial p_l} = \frac{1}{m}[p_l - (\mathbf{E}\times\boldsymbol{\mu})_l]\,.$$

[2] In the units we have chosen, i.e. $\hbar=c=1$, the momentum vector equals the wave number vector.

Because $\alpha \neq 0$, the above Dirac equation can be recast in the form

$$\Big\{ -\gamma^\mu p_\mu - \frac{1}{2\alpha^2}\gamma^\mu \bar{\Theta}_{\mu\nu} x_\nu - (1/2)\gamma^0\sigma^{12}\mu_{\rm m}\epsilon_{\mu\alpha\beta} \times \left[F^{\alpha\beta} + \frac{1}{2\alpha^2}\Theta^{\tau\sigma} p_\tau \partial_\sigma F^{\alpha\beta} \right]\gamma^\mu - m' \Big\}\psi = 0\,, \tag{29}$$

where $m' = m/\alpha$. The solution of (29) is

$$\psi = \mathrm{e}^{\mathrm{i}\hat{\varphi}_{\rm AC}}\psi_0\,, \tag{30}$$

where ψ_0 is the solution of the Dirac equation for a free particle with mass m', and $\hat{\varphi}_{\rm AC}$ stands for the AC phase on NC phase space. It has the form

$$\begin{aligned}\hat{\varphi}_{\rm AC} = &-\frac{1}{2}\gamma^0\sigma^{12}\mu_{\rm m}\int^x \varepsilon_{\mu\alpha\beta}F^{\alpha\beta}\mathrm{d}x^\mu \\ &-\frac{1}{2\alpha^2}\int^x \bar{\Theta}_{ij}x_j \mathrm{d}x_i \\ &-\frac{1}{4\alpha^2}\gamma^0\sigma^{12}\mu_{\rm m}\int^x \epsilon_{\mu\alpha\beta}\Theta^{\sigma\tau}p_\sigma\partial_\tau F^{\alpha\beta}\mathrm{d}x^\mu\,.\end{aligned} \tag{31}$$

Equation (31) is the general AC phase on non-commutative phase space. For the standard AC case, i.e. a particle moving in a pure static electric field, the AC phase reduces to

$$\begin{aligned}\hat{\varphi}_{\rm AC} &= \phi_{\rm AC} - \frac{1}{2\alpha^2}\int^x \bar{\Theta}_{ij}x_j\mathrm{d}x_i - \frac{1}{2\alpha^2}\gamma^0\sigma^{12}\mu_{\rm m} \\ &\quad\times\int^x \epsilon_{\mu 0 i}\theta\epsilon^{\alpha\beta}\left[m'v_\alpha + (\mathbf{E}\times\boldsymbol{\mu})_\alpha\right]\partial_\beta F^{0i}\mathrm{d}x^\mu \\ &= \phi_{\rm AC} - \frac{1}{2\alpha^2}\int^x \bar{\Theta}_{ij}x_j\mathrm{d}x_i + \frac{1}{2\alpha^2}\gamma^0\sigma^{12}\mu_{\rm m}\theta\epsilon^{ij} \\ &\quad\times\int^x \left[k'_j + (\mathbf{E}\times\boldsymbol{\mu})_j\right]\left(\partial_i E^2\mathrm{d}x^1 - \partial_i E^1\mathrm{d}x^2\right)\,,\end{aligned} \tag{32}$$

where $k'_j = m'v_j$, and $p_l = m'v_l + (\mathbf{E}\times\boldsymbol{\mu})_l + \mathcal{O}(\theta)$ has been applied; we omit the second order terms in θ. Equation (32) can also be written

$$\hat{\varphi}_{\rm AC} = \phi_{\rm AC} + \delta\phi_{\rm NCS} + \delta\phi_{\rm NCPS}\,, \tag{33}$$

where $\phi_{\rm AC}$ is the AC phase on commuting space (see (11)), $\delta\phi_{\rm NCS}$ is the space–space non-commuting contribution to the AC phase on NC space (see (22)), and the last term, $\delta\phi_{\rm NCPS}$, is given by

$$\begin{aligned}\delta\phi_{\rm NCPS} = &-\frac{1}{2\alpha^2}\int^x \bar{\Theta}_{ij}x_j\mathrm{d}x_i + \frac{1-\alpha^3}{2\alpha^3}\gamma^0\sigma^{12}\mu_{\rm m}\theta\epsilon^{ij} \\ &\times\int^x k_j(\partial_i E^2\mathrm{d}x^1 - \partial_i E^1\mathrm{d}x^2) \\ &+\frac{1-\alpha^2}{2\alpha^2}\gamma^0\sigma^{12}\mu_{\rm m}\theta\epsilon^{ij} \\ &\times\int^x (\mathbf{E}\times\boldsymbol{\mu})_j(\partial_i E^2\mathrm{d}x^1 - \partial_i E^1\mathrm{d}x^2)\,,\end{aligned} \tag{34}$$

which represents the non-commutativity of the momenta. The first term in $\delta\phi_{\rm NCPS}$ is a contribution purely from the non-commutativity of the momenta, the second term is a velocity dependent correction, and the third term is a correction to the vortex of the magnetic field. In the 2 dimensional non-commutative plane, $\bar{\Theta}_{ij} = \bar{\theta}\epsilon_{ij}$, and the two NC parameters θ and $\bar{\theta}$ are related by $\bar{\theta} = 4\alpha^2(1-\alpha^2)/\theta$ [9]. When $\alpha = 1$, which will lead to $\bar{\theta}_{ij} = 0$, the AC phase on NC phase space will return to the AC phase on NC space, i.e. $\delta\phi_{\rm NCPS} = 0$ and (31) and (32) will change into (18) and (21), respectively.

5 Conclusion

In this paper, we study the AC effect on both non-commutative space and non-commutative phase space. Instead of doing a tedious star product calculation, we use the "shift" method, i.e. one in which we use the fact that the star product in the Dirac equation can be replaced by a Bopp shift, and together with the shift we gave relevant definitions in (16) for NC space and (27) for NC phase space. These shifts are exactly equivalent to the star product. The additional AC phase (22) on NC space is exactly the same as in [8]; this shows the correctness of our method. Our results for the AC phase on NC phase space, especially the new term (34), which comes from the non-commutativity of the momenta, is a totally new result of this paper. The NC corrections to the AC effect may be tested in experiment in a situation of very high energy (TeV or higher). From the discussion in [8], we know that the accuracy bound of the experiment should be

$$\frac{\delta\phi_{\rm NCS}}{\phi_{\rm AC}} \leq 25\%, \quad \sqrt{\theta} \leq 10^7\ \mathrm{GeV}^{-1}\,. \tag{35}$$

Future experimental tests with high energy neutrons may lead to a better limit for the NC parameter in the AC effect.

The method we use in this paper may also be employed in other physical problems on NC space and on NC phase space. Further study of the issue will be reported on in our forthcoming papers.

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